GCSE Maths – Algebra

Solving Quadratic Inequalities

(Higher Only)

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of quadratic inequalities questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A – Higher Only

Worked Example

Solve the quadratic inequality $x^2 - 11 \geq -2x - 3$.

Present your answer on a number line.

Step 1: Rearrange the inequality so that the left-hand side of the inequality is in the form $ax^2 + bx + c$.

$$\boldsymbol{x^2 - 11 \geq -2x - 3}$$

Add $2x$ to both sides of the equation:

$$x^2 + 2x - 11 \geq -3$$

Add 3 to both sides of the equation:

$$x^2 + 2x - 8 \geq 0$$

Step 2: To find the correct region of x-values, we first draw the corresponding graph. We need to find the x-intercepts of this graph by solving the corresponding quadratic equation by factorising the quadratic equation in the inequality.

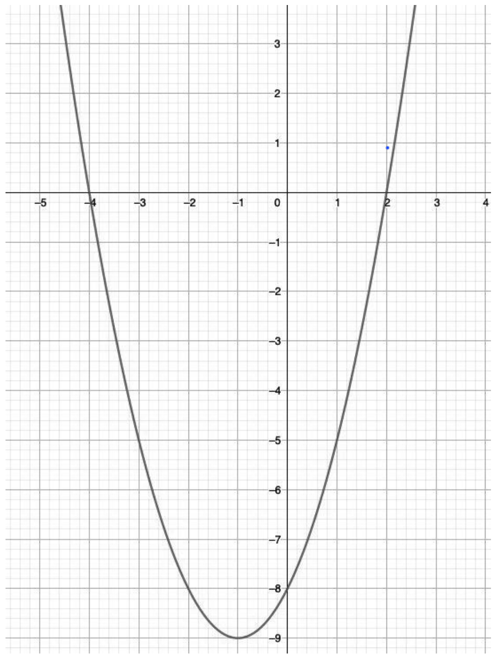
$$x^2 + 2x - 8 = 0$$
$$(x + 4)(x - 2) = 0$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \quad \text{or} \quad x = 2$$

The coordinates of the x –intercepts are $(-4,0)$ and $(2,0)$.

Step 3: Using the coordinates of the x –intercept and the y –intercept, sketch a quadratic graph equivalent to the quadratic inequality.

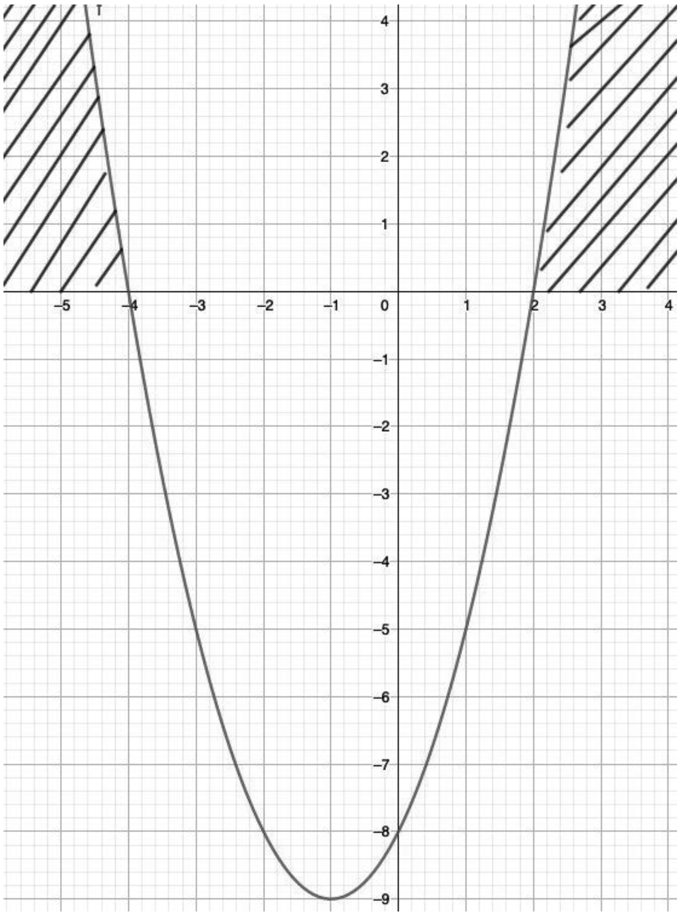
x –intercepts: $(-4,0)$ *and* $(2,0)$

y –intercept: $(0,-8)$



Step 4: Identify the required region which satisfies the quadratic inequality. Shade this region on the graph.

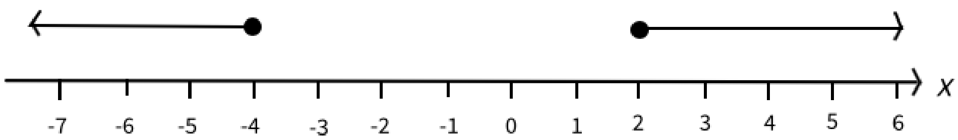
For $x^2 + 2x - 8 \geq 0$ *, the required region should be the area where the quadratic is greater than or equal to 0. In this case, the area should be the regions above the* x *–axis, as shaded in the graph below.*



Step 5: Write the answer of the required region in the form of an inequality. Since the shaded regions consist of two areas, the answer should be made up of two inequalities.

The solution is $x \leq -4$ *or* $x \geq 2$.

Step 6: Present the answer in a number line.



In the number line, we use solid filled circles to denote the fact that $x = -4$ *and* $x = 2$ *is included in the values for* x.

Guided Example

Solve the inequality $2x^2 - 4x - 4 \leq 6x + 8$.

Present your answer in a number line.

Step 1: Rearrange the inequality so that the left-hand side of the inequality is in the form $ax^2 + bx + c$.

$$2x^2 - 4x - 4 \leqslant 6x + 8$$
$-6x$
$$2x^2 - 10x - 4 \leqslant 8$$
-8
$$2x^2 - 10x - 12 \leqslant 0$$

Step 2: To find the correct region of x-values, we first draw the corresponding graph. We need to find the x-intercepts of this graph by solving the corresponding quadratic equation by factorising the quadratic equation in the inequality.

$$2x^2 - 10x - 12 = 0$$
$\div 2$
$$x^2 - 5x - 6 = 0$$
Factorise
$$(x-6)(x+1) = 0$$

$(x-6) = 0$ or $(x+1) = 0$

$x = 6$ or $x = -1$

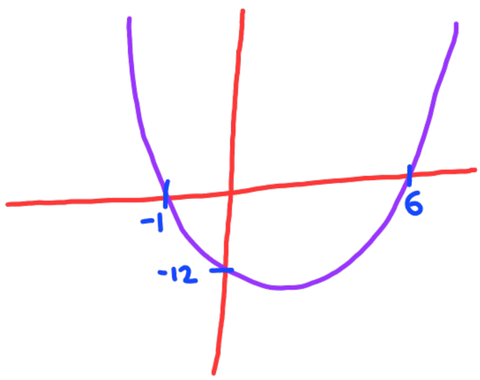
Co-ordinates of x-intercept
$(-1, 0)$, $(6, 0)$
- where the curve crosses the x-axis

Step 3: Using the coordinates of the x –intercept and the y –intercept, sketch a quadratic graph equivalent to the quadratic inequality

when $x = 0$

$2x^2 - 10x - 12 \rightarrow 2(0)^2 - 10(0) - 12$
$= 0 - 0 - 12$
$= -12 \rightarrow$ y-intercept : $(0, -12)$
- where the curve crosses the y-axis

x-intercepts :
$(6, 0)$, $(-1, 0)$

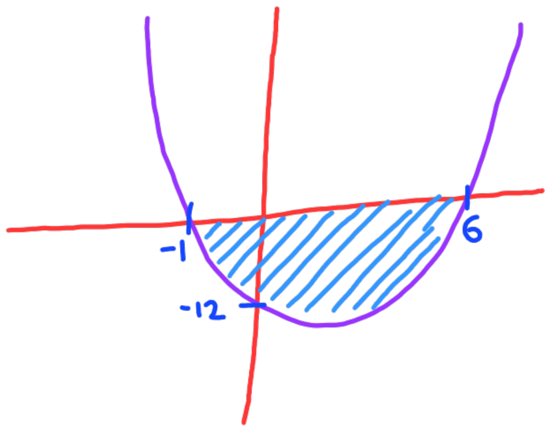


Step 4: Identify the required region which satisfies the quadratic inequality. Shade this region on the graph.

$2x^2 - 10x - 12 \leq 0$

The quadratic is smaller than 0.

Therefore : region below the x axis.

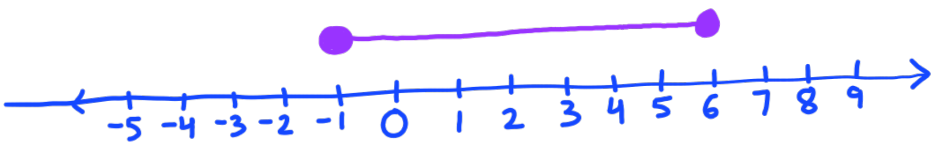


Step 5: Write the answer of the required region in the form of an inequality. Since the shaded regions consist of two areas, the answer should be made up of two inequalities.

As shown above x is between -1 and 6

$-1 \leq x \leq 6$

Step 6: Present the answer in a number line.



The circles are filled because -1 and 6 are included in the values for x.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

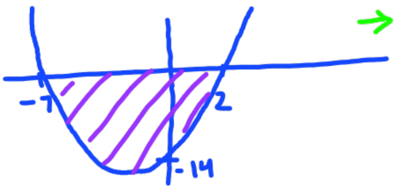
1. Solve the following inequalities and present your answer in a number line.

a) $3x^2 + 2x < 14 + 2x^2 - 3x$

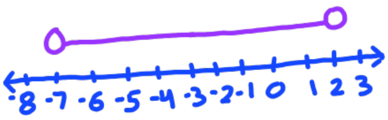
$3x^2 + 2x < 14 + 2x^2 - 3x$ $-2x^2$
$x^2 + 2x < 14 - 3x$ $+3x$
$x^2 + 5x < 14$ -14
$x^2 + 5x - 14 < 0$

Factorisation
$x^2 + 5x - 14 = (x+7)(x-2)$
$(x+7) = 0$ $(x-2) = 0$
$x = -7$ $x = 2$
x intercepts : $(-7, 0)$, $(2, 0)$

when $x = 0$
$x^2 + 5x - 14 = 0^2 + 5(0) - 14 = -14$
y-intercept : $(0, -14)$



$\rightarrow$ $x^2 + 5x - 14 < 0$
$\therefore$ below x axis
x between
-7 and 2 $\rightarrow$ $-7 < x < 2$



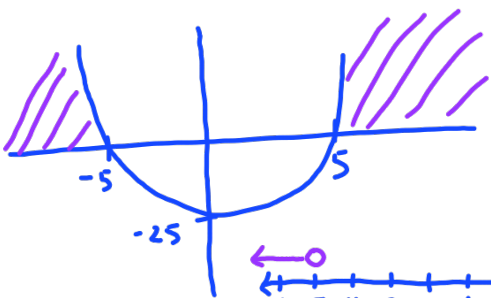
unfilled circles because -7 and 2 are not included in the values for x

b) $x^2 + 5 > 30$

$x^2 + 5 > 30$ -30
$x^2 - 25 > 0$ Factorise
$(x+5)(x-5) > 0$

Factorisation
$(x+5)(x-5) = 0$
$x = -5$ $x = 5$
x-intercepts : $(-5, 0)$, $(5, 0)$

when $x = 0$
$x^2 - 25 = 0^2 - 25 = -25$
y-intercept : $(0, -25)$



$\rightarrow$ $x^2 - 25 > 0$
$\therefore$ Above x-axis
$x < -5$ $x > 5$

unfilled circles because -5 and 5 are not included in the values for x

c) $-2x^2 + 4 \geq x^2 + 9x - 8$

$-x^2$: $-2x^2 + 4 \geqslant x^2 + 9x - 8$

$-9x$: $-3x^2 + 4 \geqslant 9x - 8$

$+8$: $-3x^2 - 9x + 4 \geqslant -8$

$-3x^2 - 9x + 12 \geqslant 0$

when $x = 0$
$-3x^2 - 9x + 12$
$= -3(0)^2 - 9(0) + 12$
$= 0 - 0 + 12$
$= 12$ y-intercept: $(0,12)$

$-3x^2 - 9x + 12 = 0$
$-x^2 - 3x + 4 = 0$
$-(x^2 + 3x - 4) = 0$
$-(x+4)(x-1) = 0$
$(-x-4)(x-1) = 0$

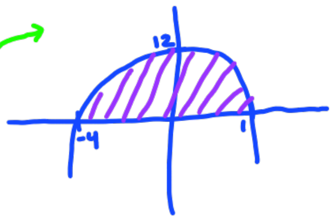
$-x - 4 = 0$ $\quad x - 1 = 0$
$-x = 4$ $\quad x = 1$
$x = -4$

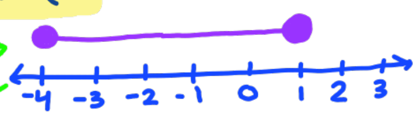
x-intercepts: $(-4,0), (1,0)$

Quadratic
$\underline{-3}x^2 - 9x - 12$
coefficient of $x^2 = \underline{-3}$ negative → shape of graph:

Filled circles because -4 and 1 are included in the x-values

$-3x^2 - 9x + 12 \geqslant 0$
∴ Above x-axis
$-4 \leqslant x \leqslant 1$





d) $x^2 + 12 \geq -7x + 2$

$+7x$: $x^2 + 12 \geqslant -7x + 2$

-2: $x^2 + 7x + 12 \geqslant 2$

Factorise: $x^2 + 7x + 10 \geqslant 0$

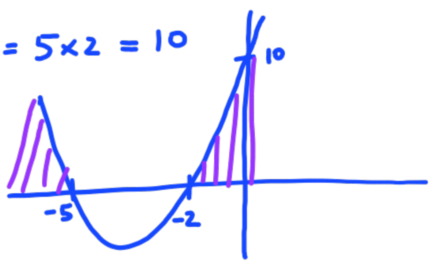
$(x+5)(x+2) \geqslant 0$

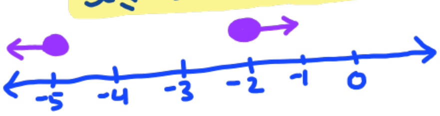
To find x-intercepts
$(x+5)(x+2) = 0$
$(x+5) = 0 \quad (x+2) = 0$
$x = -5 \quad x = -2$

x-intercepts: $(-5,0), (-2,0)$

when $x = 0$
$(x+5)(x+2) = (0+5)(0+2) = 5 \times 2 = 10$
y-intercept $= (0,10)$

$x^2 + 7x + 10 \geqslant 0$
∴ above the x-axis
$x \leqslant -5 \quad x \geqslant -2$





Filled circles because -5 and -2 are included in the x-values